# Nonlinear Adaptive Control for High Angle of Attack Flight 

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This paper presents a nonlinear methodology for the control of a high angle of attack aircraft, in particular, a modified F-18 aircraft. As a modern combat aircraft demands better maneuverability and performance over domains which include high angles of attack, research in high angle of attack is presently at an advanced stage. An adaptive controller is developed to maneuver an aircraft at a high angle of attack even if the aircraft is required to fly over a highly roonlinear flight regime. The adaptive controller presented in this paper is based on nonlinear prediction models, and can be constructed to minimize the given cost function or the difference of a described Lyapunov function with respect to the control input at each step. A controller uses system identification parameters to calculate a command signal so that the output of system follows the reference trajectory. The control is calculated to let system follow the reference trajectory under some constraints. This paper shows that nonlinear adaptive control can be utilized effectively to control high performance aircraft such as the F-18 aircraft for rapid maneuvers with large changes in angle of attack.

Key Words: Adaptive Control, Nonlinear Prediction Model, Angle of Attack, Maneuver

## Nomenclature

$L F$ : Lyapunov function
$P \quad$ : Covariance matrix
$a \quad$ : Angle of attack
$\alpha_{\text {ref }} \quad:$ Reference trajectory of angle of attack
$\alpha_{\mathrm{cmd}}:$ Command signal of angle of attack
$\delta_{h} \quad:$ Elevator deflection
$\delta_{\mathrm{h}}^{\mathrm{h} m \alpha} \mathrm{a}$ : Command signal of elevator deflection
$\delta_{v_{c m d}}$ : command signal of thrust vector angle
$\partial_{x} \quad:$ Thrust vector angle between $T_{x}$ and $T_{z}$
$\rho_{\mathrm{i}} \quad:$ Weighting factor in performance index
$\Phi \quad:$ Regression vector in system identification
$\hat{\theta} \quad$ : Parameters to be estimated

## 1. Introduction

In the design of flight control systems, flight
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control is driven by the nonlinear and timevarying nature of aircraft dynamics. The conventional solution to this problem is to perform trim point designs for a large set of trim flight conditions and construct a gain schedule by interpolating gains with respect to flight conditions(Halyo and Moerder, 1989). A conventional control law(constant gain)may not meet high performance requirements in the presence of large changes in the operating parameters, but a variable gain control law approach has been developed to provide a class of controller which is highly maneuverable with high performance over a wide range of operating conditions(Ostroff, 1989, 1992). Ostroff's approach is to extend the operationg range of the control law over the flight regime while continuing to use established linear control design and analysis techniques. In his approach, the system equations were constructed as linear models even if the system equations change according to flight conditions. Whenever flight conditions change, the variable gain output feedback is applied. In the other approach proposed by Buffington, Sparks, and

Banda(1993), the control law is based on a linear $\mathbf{H}_{\infty}$ design in conjunction with trim-state linearized dynamics and an appropriate nonlinear gain scheduled according to dynamic pressure variation. While neither of the two approaches are nearly minimum time maneuvers, they probably represent the best controllers based primarily on linear design methodology in conjunction with somewhat ad-hec nonlinear corrections. This paper shows that nonlinear control can be utilized effectiyely to control high performance aircraft such as F-18 aircraft for rapid maneuvers with large changes in ange of attack. Nonlinear feedback controllers that were generated in conjunction with a linear model reference without multiple regression terms failed for certain highalpha maneuver, but with added nonlinear reference terms, they lead to successful control. This paper, however, indicates that the nonlinear feedback controller generated in conjunction with a higher-order (more delay terms) linear model reference also is quite effective. To improve performance of a nonlinear aircraft system, and to reduce the response time of states in maneuvering of aireraft at high angle of attack, adaptive approach has been used. The purpose for adaptive control is to provide a mechanism to account for changes in the system that is to be controlled. The idea of adaptive model reference control is to identify the system. A model system generates a desired reference trajectory. Then, a controller uses this information to calculate a command signal so that the output of the system follows the reference trajectory. A block diagram of the model reference adaptive controller is shown in Fig. 1. The control is calculaed such that the system follows the reference trajectory, and such that the control signal remains within its


Fig. 1 Block diagram of adaptive control
constraints.

## 2. Dynamic Equation

The aircraft model described in this section is based on a modified version of the F-18 aircraft. The controllers consist of the elevator and thrust vectoring. The elevator are useful at normal flight conditions, while the thrust vectoring is useful at high angle of attack, and at low dynamic pressure operating conditions. The mathematical structure of the aerodynamic coefficients are based on the wind tunnel test data(Cao et al, 1992). The aerodynamic coefficients are considered to be functions of the elevator deflection as well as angle of attack, Mach number, total speed, and altitude. The nonlinearities of both the aerodynamics and thrust vectoring are preserved in the model. This paper mentions only the longitudinal motion described by angle of attack, pitch rate, pitch angle, and total speed(Cho, 1994).

### 2.1 Actuator dynamics

The input dynamics were described by three states-thrust magnitude ( T ), thrust vectoring angle $\left(\delta_{\mathrm{v}}\right)$, and elevator angle ( $\delta_{\mathrm{h}}$ ). Dynamic equations were extended by including the actuator dynamics for $\delta_{\mathrm{h}}, \delta_{\mathrm{v}}$, and T. Dynamics of T is assumed to be linear of the first order with time constant 1 second. Time constants for $\delta_{\mathrm{h}}$ and $\delta_{\mathrm{v}}$ are $1 / 30$ sec., but the rate of change is constrained to be smaller than $40 \mathrm{deg} . / \mathrm{sec}$. The saturation was modeled in a smooth way to make the gradient calculations possible. The actuators were modeled as

$$
\begin{align*}
& \frac{d \delta_{\mathrm{h}}}{d t}=g\left(-30 \delta_{\mathrm{h}}+30 \delta_{\mathrm{hcmd}}\right)  \tag{1}\\
& \frac{d \delta_{\mathrm{v}}}{d t}=g\left(-30 \delta_{\mathrm{v}}+30 \delta_{\mathrm{v} \mathrm{cmd}}\right)  \tag{2}\\
& \frac{d T}{d t}=-T+T_{\mathrm{cmd}} \tag{3}
\end{align*}
$$

where a saturation function $g$ is modeled as

$$
g(x)= \begin{cases}40-\exp (-x-39) & \text { if } x>39  \tag{4}\\ x & \text { if } 39 \geq x \geq-39 \\ -40+\exp (x+39) & \text { if } x<-39\end{cases}
$$

The range of the elevator angle and thrust vector-
ing angle is limited according to the following:

$$
\begin{align*}
& -24^{\circ} \leq \delta_{\mathrm{h}} \leq 10.5  \tag{5}\\
& -20^{\circ} \leq \delta_{\mathrm{v}} \leq 20^{\circ} \tag{6}
\end{align*}
$$

The magnitude of thrust is limited according to the following equation:

$$
\begin{equation*}
0 \leq T \leq 80 \mathrm{kN} \tag{7}
\end{equation*}
$$

## 3. Frediction Model for a Modified F-18 Aircraet

An approach to formulate a prediction model is to use a more complex nonlinear representation. There are several standard input-output modeling techniques for nonlinear systems in both discrete and continuous time settings. In this work, the time-series approach is discussed. The nonlinear time series expresses future values of output as a nonlinear function of a finite number of past values of output and of control. Even if the physical dynamics of an aircraft are well known and is easily expressible in the state space form, there are significant reasons to use input-
output black-box-type modeling as an alternative approach. The main problem arises from the aerodynamic stability derivatives. They are complex nonlinear functions of angle of attack, Mach number, and altitude If these relationships are entered into the state space model, it appears so compllcated that its usefulness for on-line control generation becomes quite doubtful. Furthermore, the exact form of the dependencies for stability derivatives on state variables is not known. Aerodynamic coefficients are functions of angle of attack, Mach number, elevator angle, and altitude. Specifically, coefficient values depend on angle of attack. A slightly more complex nonlinear prediction model for the aircraft is considered by adding quadratics and cubics in angle of attack which naturally would better fit the aerodynamic parameters. Thrust vector must be also considered for rapid high angle of attack maneuvers. A nonlinear prediction model proposed in this section, including the thrust vector, was developed as follows:

$$
\begin{align*}
& \hat{\alpha}=\Phi^{\mathrm{T}}(t) \hat{\theta}(t-1)  \tag{8}\\
& \Phi^{\mathrm{T}}(i)=\left[\alpha(t-2) q(t-2) \alpha(t-3) q(t-3) \alpha(t-4) q(t-4) \delta_{\mathrm{h}}(t-1) \delta_{\mathrm{v}}(t-1) \delta_{\mathrm{h}}(t-2)\right. \\
& \quad \delta_{\mathrm{v}}(t-2) \hat{\delta}_{\mathrm{h}}(t-3) \\
& \delta_{\mathrm{v}}(t-3) \alpha(t-2) \delta_{\mathrm{h}}(t-1) \alpha(t-3) \delta_{\mathrm{h}}(t-2) \alpha(t-2) q(t-1) \alpha(t-3) q(t-2) \alpha(t-2) \delta_{\mathrm{v}}(t-1) \\
& \left.\alpha(t-3) \delta_{\mathrm{v}}(t-2) \alpha^{2}(t-2) \alpha^{2}(t-3) \alpha^{3}(t-2) \alpha^{2}(t-2) q(t-2) \alpha^{2}(t-2) \delta_{\mathrm{h}}(t-1) \alpha^{2}(t-2) \delta_{\mathrm{h}}(t-1)\right] \tag{9}
\end{align*}
$$

The choice of elements of the regressor vector is motivated by the fact that nonlinearities in the short period dynamics are associated with angle of attack. Also it is recognized that due to the highly nonlinear nature of the aircraft dynamics it is probably impossible to fit a black-box-type model describing the plant's dynamics accurately in the whole range of flight condition. Instead, it is more practical to fit a simple approximate model including square and cubic terms of angle of attack, thrust vectoring, and coupling term between angle of attack and control inputs.

### 3.1 Parameter estimation

The recursive-least-squares (RLS) algorithm is the most popular on-line parameter estimation algorithm. The main idea is to obtain model parameter estimates which, in a least squares sense, minimize the difference between the actual
output, $\mathrm{y}(\mathrm{t})$, and its value predicted by the model.
This leads to the recursive least squares algorithm with a variable forgetting factor (Goodwin, 1984) as follows.
Parameter vector update law:

$$
\begin{align*}
\bar{\theta}(t)= & \bar{\theta}(t-1)+K(t)[y(t) \\
& \left.-\hat{\theta}^{\mathrm{T}}(t-1) \Phi(t)\right] \tag{10}
\end{align*}
$$

Gain update:

$$
\begin{equation*}
K(t)=\frac{P(t-1) \Phi(t)}{\lambda+\Phi^{\mathrm{T}}(t) P(t-1) \Phi(t)} \tag{11}
\end{equation*}
$$

Covariance matrix update:

$$
\begin{align*}
P(t)= & \frac{1}{\lambda}(P(t-1) \\
& \left.-\frac{P(t-1) \Phi(t) \Phi^{\mathrm{T}}(t) P(t-)}{\lambda+\Phi^{\mathrm{T}}(t) P(t-1) \Phi(t)}\right) \tag{12}
\end{align*}
$$

The basic RLS algorithm with $\lambda=1$ has several important properties. First the least-squares algorithm has a fast convergence rate. Also, the stability of the RLS algorithm combined with direct
and indrect adaptive control is well understood(Goodwin, 1984). The main disadvantage with the basic RLS algorithm is that the covariance matrix gradually decays to a small value and therefore the algorithm does not its adaptivity to adequately track time varying systems. The covariance matrix in the RLS algorith$m$ tends towards zero which causes the adaptation to turn off. This is undesirable in the case where the parameters are time varying. Several modifications of two types have been made to the RLS algorithm to correct this problem. The first idea is to manipulate the covariance matrix directly. The second type of modifications is the inclusion of a forgetting factor as is discussed next.

### 3.2 RLS with constant trace and scaling

Sripada and Fisher(1987) have proposed the following four modification to the basic least squares algorithm.
(1) Normalization
(2) Scaling
(3) Constant trace through a variable forgetting factor
(4) An information content based on criterion for turning adaptation on or off.
The importance of normalization and forgetting factor has already been discussed(Goodwin, 1984). The modification with respect to scaling is concerned with improving the numerical properties of the algorithm but has no effect on the convergence properties of the algorithm. Property (3) concerns updating of the covariance matrix the forgetting factor $\lambda(t)$ is selected so that the trace of the covariance matrix is constant. The following choice of ensure that and that trace $\mathrm{P}($. is constant

$$
\begin{align*}
\lambda(t)= & \mathbf{I}-\frac{1}{2}\left(1+r-\left[(1+r)^{2}\right.\right. \\
& \left.\left.-4 \frac{\left\|P_{\mathrm{s}}(t-1) \phi(t)_{\mathrm{ns}}\right\|^{2}}{\operatorname{tr}\left(P_{\mathrm{s}}(t-1)\right)}\right]^{\frac{1}{2}}\right) \tag{13}
\end{align*}
$$

where

$$
\begin{equation*}
r=\phi(t)_{\mathrm{ns}}^{\mathrm{T}} P(t-1)_{\mathrm{s}} \phi(t)_{\mathrm{ns}} \tag{14}
\end{equation*}
$$

$\mathrm{P}_{\mathrm{s}}($.$) corresponds to the scaled covariance matrix$ and $\phi(t)_{\mathrm{ns}}$ corresponds to the normalized and scaled regressor. The constant trace of $P_{s}($.
ensures an upper bound on the maximum eigenvalue. The modification of Eq. (12) determines the extent of discounting of old information in the current update of $P_{s}($.

### 3.3 Reference model

In this section the reference model is an intermediate step that allows the system to follow the command signal while meeting a variety of design criteria. The proposed approach used feedback from the real plant to improve the reference trajectory (Collins, 1993). The class of models for the reference trajectory that were investigated are simply filters that use the past values of the states. Thus, the reference model has no internal states of its own. For the complete system, a second-order filter was found to be sufficient to get excellent performance. A general second-order filter is described in Eq. (15). The parameters of the reference trajectory were not fixed but varied according to the gain schedule listed in the Table 1. The second reference trajectory is

$$
\begin{align*}
\alpha_{\mathrm{ref}}(t)= & C_{1}(\mathrm{c}) \alpha(t-1)-C_{2}(\mathcal{c}) \alpha(t-2) \\
& +\left(1-C_{1}(\mathcal{c})+C_{2}(\varsigma)\right) \alpha_{\mathrm{cmd}}(t) \tag{15}
\end{align*}
$$

where

$$
\begin{equation*}
\mathfrak{c}=\left|\alpha_{\mathrm{cmd}}(t-1)-\alpha(t-1)\right| \tag{16}
\end{equation*}
$$

The values were chosen such that all but the first filter correspond to a constant percent overshoot with diticient rise times.

Table 1 constants for Equation(15)

|  | $\mathrm{C}_{1}(\mathrm{C})$ | $\mathrm{C}_{2}(\mathrm{C})$ |
| :---: | :---: | :---: |
| $0 \leq ¢<1$ | 1.7600 | 0.7743 |
| $1 \leq ¢<2$ | 1.7215 | 0.7517 |
| $2 \leq ¢<3$ | 1.7563 | 0.7796 |
| $3 \leq ¢<4$ | 1.7734 | 0.7937 |
| $4 \leq ¢<6$ | 1.7904 | 0.8079 |
| $6 \leq ¢<8$ | 1.8073 | 0.8221 |
| $8 \leq ¢<10$ | 1.82241 | 0.8365 |
| $10 \leq c ¢<15$ | 1.8407 | 0.8509 |
| $15 \leq ¢<25$ | 1.8572 | 0.8655 |
| $25<¢$ | 1.8736 | 0.8801 |

## 4. Control Caculation

Our interest is to design controller which performs or meets several goals. First an most importantly, the control values are calculated such that the angle of attack of the aircraft accurately follows the reference model. The control values are also calculated such that the thrust vectoring returns to zero if it is no longer needed and a certain amount of smoothness is desired for the control signals. The input dynamics were described by three states-thrust magnitude, thrust vectoring angle, and elevator angle. The elevator and the thrust vectoring angle include a velocity limiter of 40 degrees per second and 80 degrees per second, respectively. The range of the elevator angle and the thrust vectoring angle are shown in the Eqs. (5) ~(6), respectively. The magnitude of thrust is scheduled like the Eq. (7). Two types of
control law are considered in this section. First, one-step-ahead prediction controller is calculated. The following cost function is defined for control law calculation.

$$
\begin{align*}
J= & \frac{1}{2} \rho_{1}\left(\alpha_{\mathrm{ref}}(t+1)-a(t+1)\right)^{2}+\frac{1}{2} \rho_{2} \\
& \left(\delta_{\mathrm{hemd}}(t)-\delta_{\mathrm{hcmd}}(t-1)\right)^{2}+\frac{1}{2} \rho_{3}\left(\delta_{\mathrm{v} \mathrm{cmd}}(t)\right. \\
& \left.-\delta_{\mathrm{v}_{\mathrm{cmd}}}(t-1)\right)^{2}+\frac{1}{2} \rho_{4}\left(\delta_{\mathrm{v}_{\mathrm{cmd}}}(t)\right)^{2} \tag{17}
\end{align*}
$$

The nonlinear prediction model for the angle of attack described in the Eqs. (8) - (9) can be rewritten as follows.

$$
\begin{align*}
\bar{\alpha}(t+1)= & \left(a_{11}(t)+a_{12}(t) \alpha(t)\right. \\
& \left.+a_{13}(t) \alpha^{2}(t)\right) \delta_{\mathrm{hcma}}(t) \\
& +\left(a_{14}(t)+a_{15}(t) \alpha(t)\right. \\
& \left.+a_{16}(t) \alpha^{2}(t)\right) \delta_{v_{\mathrm{cmd}}}(t) \\
& +\bar{\phi}(t+1) \bar{\theta}(t) \tag{18}
\end{align*}
$$

where

$$
\begin{align*}
\bar{\phi}^{\mathrm{T}}(t+1)= & {\left[\alpha(t-1) q(t-1) \alpha(t-2) q(t-2) \alpha(t-3) q(t-3) \delta_{\mathrm{hema}}(t-1) \delta_{\mathrm{vcma}}(t-1) \delta_{\mathrm{nema}}\right.} \\
& (t-2) \delta_{\mathrm{vcma}}(t-2) \alpha(t-2) \delta_{\mathrm{hema}}(t-1) \alpha(t-2) q(t-1) \alpha(t-2) q(t-2) \\
& \left.\alpha(t-2) \delta_{\mathrm{vemd}}(t-1) \alpha^{2}(t-1) \alpha^{3}(t-1) \alpha^{3}(t-2) \alpha^{2}(t-1) q(t-1)\right]  \tag{19}\\
\theta^{\mathrm{T}}(t)= & {\left[a_{11}(t), a_{12}(t), a_{13}(t), a_{14}(t), a_{15}(t) a_{16}(t), \bar{\theta}^{\mathrm{T}}(t)\right] }  \tag{20}\\
\Phi^{\mathrm{T}}(t+1)= & {\left[\delta_{\mathrm{ncma}}(t) \alpha(t) \delta_{\mathrm{hcma}}(t) \alpha^{2}(t) \delta_{\mathrm{ncma}}(t) \delta_{\mathrm{vcma}}(t) \alpha(t) \delta_{\mathrm{vema}}(t) \alpha^{2}(t) \delta_{\mathrm{vcma}}(t) \bar{\phi}^{\mathrm{T}}(t+1)\right] } \tag{21}
\end{align*}
$$

Equation (18) can be rewritten as follows.

$$
\begin{align*}
\hat{\alpha}(t+1)= & a(t) \delta_{\mathrm{hcmd}}(t)+b(t) \delta_{\mathrm{v}_{\mathrm{cmd}}}(t) \\
& +\bar{\phi}^{\mathrm{T}}(t+1) \bar{\theta}(t) \tag{22}
\end{align*}
$$

where

$$
\begin{align*}
& a(t)=a_{11}(t)+a_{12}(t) \alpha(t)+a_{13}(t) \alpha^{2}(t)  \tag{23}\\
& b(t)=a_{14}(t)+a_{15}(t) \alpha(t)+a_{16}(t) \alpha^{2}(t) \tag{24}
\end{align*}
$$

Taking the derivative of J with respect to the control yields

$$
\begin{align*}
& \frac{d J}{d \delta_{\mathrm{ncmd}}(t)}=\rho_{1}\left[\alpha_{\mathrm{ref}}(t+1)-\alpha(t+1)\right](-a(t))+\rho_{2}\left[\delta_{\mathrm{hcmd}}(t)-\delta_{\mathrm{hcmd}}(t-1)\right]  \tag{25}\\
& \frac{d J}{d \delta_{\mathrm{v} \mathrm{cmd}}(t)}=\rho_{1}\left[\alpha_{\mathrm{ref}}(t+1)-\alpha(t+1)\right](-b(t))+\rho_{3}\left[\delta_{\mathrm{v}_{\mathrm{cmd}}}(t)-\delta_{\mathrm{v} \mathrm{cmd}}(t-1)\right]+\rho_{4}\left[\delta_{\mathrm{vcmd}}(t)\right] \tag{26}
\end{align*}
$$

Consequently, the external control command yields

$$
\begin{align*}
& \delta_{\mathrm{hcmd}}(t)=\frac{\rho_{1}\left(\rho_{3}+\rho_{4}\right) a \eta+\left(\rho_{1} b^{2}+\rho_{3}+\rho_{4}\right) \rho_{2} \delta_{\mathrm{hemd}}(t-1)-\rho_{1} \rho_{3} a b \delta_{\mathrm{vcmd}}(t-1)}{a^{2} \rho_{1}\left(\rho_{3}+\rho_{4}\right)+b^{2} \rho_{1} \rho_{2}+\rho_{2}\left(\rho_{3}+\rho_{2}\right)}  \tag{27}\\
& \delta_{\mathrm{hcmd}}(t)=\frac{\rho_{1} \rho_{2} b \eta+\left(\rho_{1} a^{2}+\rho_{2}\right) \rho_{3} \delta_{\mathrm{vcmd}}(t-1)-\rho_{1} \rho_{2} a b \delta_{\mathrm{hcmd}}(t-1)}{a^{2} \rho_{1}\left(\rho_{3}+\rho_{4}\right)+b^{2} \rho_{1} \rho_{2}+\rho_{2}\left(\rho_{3}+\rho_{2}\right)} \tag{28}
\end{align*}
$$

where

$$
\begin{equation*}
\eta=\alpha_{\mathrm{ref}}(t+1)-\bar{\phi}^{\mathrm{r}}(t+1) \bar{\theta}(t) \tag{29}
\end{equation*}
$$

To include the velocity and magnitude limits in the control calculation, two extra conditions are
added. The first condition requires that $\delta_{\mathrm{v}_{\mathrm{cma}}}(t)$ be recalculated if $\delta_{\mathrm{hcmd}}(t)$ has reached the magnitude limit. The second condition requires that $\delta_{\mathrm{vamd}}(t)$ be recalculated if $\delta_{\mathrm{hemd}}(t)$ is a value requiring 40 degrees per second. $\delta_{\mathrm{v}_{\mathrm{c}} \mathrm{md}}(t)$ is recalculated as follows:

$$
\begin{align*}
& \delta_{\mathrm{vamd}}(t)=\frac{\eta-a(t) \delta_{\mathrm{hcmd}}(t)}{b(t)} \\
& V_{t}=\left(\alpha_{\mathrm{ref}}(t+1)-\bar{a}(t+1)\right)^{\mathrm{T}} P_{\mathrm{L}}\left(\alpha _ { \mathrm { ref } } \left(t+1(-\widehat{\alpha}(t+1))+u_{\mathrm{t}}^{\mathrm{T}} R u_{\mathrm{t}}\right.\right. \tag{31}
\end{align*}
$$

where $P_{L}$ and $R$ are positive definite and symmetric respectively. $u_{\mathrm{t}}=\left[\delta_{\mathrm{h}_{\mathrm{cmd}}}(t), \delta_{v_{\mathrm{c} m d}}(t)\right]^{\mathrm{T}}$. The difference of a Lyapunov function candidate is given by

$$
\begin{equation*}
\Delta V_{\mathrm{t}}=V_{\mathrm{t}}-V_{\mathrm{t}-1} \tag{32}
\end{equation*}
$$

In this section, the main objective is to find a controller which minimizes the difference of a Lyapunov function candidate such that $\Delta V_{\mathrm{t}}<0$. Define $\bar{B}$ as follows:

$$
\begin{equation*}
\bar{B}=[a(t), b(t)] \tag{33}
\end{equation*}
$$

Next take the derivative of Eq. (32) with respect to the controls, and set it to 0

$$
\begin{equation*}
\frac{\partial \Delta V_{\mathrm{t}}}{\partial u_{\mathrm{t}}}=0 \tag{34}
\end{equation*}
$$

Now the controls become

$$
\begin{equation*}
u_{\mathrm{t}}=\left(\bar{B}^{\mathrm{T}} P_{\mathrm{L}} \bar{B}+R\right)^{-1} \bar{B}^{\mathrm{T}} P_{\mathrm{L}} \eta \tag{35}
\end{equation*}
$$

Similarly, the velocity and magnitude limits are considered. After the control values have been calculated they are limited by 40 degrees per second for $\delta_{\text {hemd }}(t)$ and by 80 degrees per second for $\delta_{\mathrm{vama}}(t)$.

## 5. Simulation

In this section, longitudinal motions were analyzed and simulated with an integration time step of 0.01 second. Two control signals, elevator angle and thrust vectoring angle, are used with scheduled thrust magnitude. The maneuvers presented here were done at $5,000 \mathrm{~m}$. A dotted line in Fig. 2 displays the command signal from 5 degrees, to 60 degrees, 35 degrees, and to 5 degrees in case of Maneuver One, while a dotted line of Fig. 6 displays the command signal from 5 degrees, to 35 degrees, and to 85 degrees in case of

After the control values have been calculated, they are limited by 40 degrees per second for $\delta_{\text {hamd }}$ $(t)$ and by 80 degrees per second for $\delta_{v_{\mathrm{cmo}}}(t)$. Second, control law based on a Lyapunov function is considered. A Lyapunov function candidate is given as follows.
maneuver Two. Trim conditions of nonlinear longitudinal motion are given next. Angle of Attack, pitch rate, pitch angle, total speed are given by 5 degrees, 0 degrees $/ \mathrm{sec}$., 6.3 degrees, and $150 \mathrm{~m} / \mathrm{sec}$, respectively. Elevator angle, thrust vector angle, and magnitude of thrust is given by -0.8 degrees, 0 degrees, and 13.34 kN . The reference trajectory of angle of attack is generated by a second-order filter described in Eq. (15). Another command trajectory, magnitude of thrust, is given as the dotted line in Fig. 5. The weighting factors in the cost function are given by Table 2.
In the case of a nonlinear prediction model, gain schedules of weighting factor are applied at 35 degrees of angle of attack. The weighting factors in the Lyapunov function are given as follows.

$$
P_{\mathrm{L}}=9984
$$

$$
R=\left[\begin{array}{ll}
2.68 & 2.01 \\
2.01 & 1.51
\end{array}\right]
$$



Fig. 2 Angle of attack in case of maneuver one and prediction controller.

Table 2 The cost function weighting factors.

|  | $\rho_{1}$ | $\rho_{2}$ | $\rho_{3}$ | $\rho_{4}$ |
| :---: | :---: | :---: | :---: | :---: |
| Maneuver One in case of | 94.21 | 0.001 | 0.001 | 0.0975 |
| alpha $=35$ degrees | 95 | 0.0001 | 0.01 | 0.0001 |
| Maneuver Two in case of | 95 | 0.001 | 0.001 | 0.1 |
| alpha $=\mathbf{3 5}$ degrees | 97 | 0.0001 | 0.1 | 0.0002 |



Fig. 3 Elevator angle in case of maneuver one and prediction controller.


Fig. 4 Thrust vector angle in case of maneuver one and prediction controller.


Fig. 5 Magnitude of thrust vector in case of maneuver one and prediction controller.

### 5.1 Simulation results

In Figs. 2~8, the longitudinal motion of a modified F-18 aircraft is demonstrated successfully by accurate computer simulations. The main purpose of these adaptive controllers is to cortrol the angle of attack as fast as possible to follow the command trajectory of the angle of attack. Similarly Ostroff investigated the maneuver by using numerous trim-state linearization studies accompanied by schedulcd variable gain in a PIF controller (Ostroff, 1992) for the case of Maneuver One. The angle of attack trajectories obtained by the one-step-ahead prediction controller are shown in Fig. 2. The character of the response for Maneuver One is similar to the response reported by Ostroff (1992). The angle of attack reaches 55 degrees in approximately 2.0 seconds with a settling time to 60 degrees of about 3 seconds. For Maneuver Two, with a Lyapunov function controller, the angle of attack changed from 35 degrees to 80 degrees in approximately 2.5 sec -


Fig. 6 Angle of attack in case of maneuver two and LF controller.


Fig. 7 Elevator angle in case of maneuver two and L.F controller.


Fig. 8 Thrust vector angle in case of maneuver two and LF controller.
onds and the settling time to 85 degrees of angle of attack took about 3.5 seconds. In the case of a $\mathrm{H} \infty$ controller (Buffington et al, 1993), an angle of attack change from 10 to 20 degrees took about 3 seconds with a rise time of 1 second. As a benchmark for comparison, the time optimal control (with a limitation of 40 degrees per second on the thrust vectoring) reaches 55 degrees in about 1.8 seconds(Mohler et al, 1993).

## 6. Conclusions

In this paper, an effective nonlinear control design methodology using an adaptive control law has been presented. The control law was applied to a highly nonlinear maneuverable high performance aircraft. The character of the response for Maneuver One is similar to the response reported by Ostroff (1992). The one-step ahead prediction adaptive controller provided a somewhat faster response when it was compared
with the Ostroff's result the variable gain approach (Ostroff, 1992). With physical constraints on the magnitude and rate of control changes, bang-bang control control was applied in regions which the real angle of attack and command trajectory of angle of attack are big different. Particularly, the nonlinear adaptive controller is more effective than the linear adaptive controller as angle of attack is increased (Cho, 1994). This paper shows that nonlinear control (i. e., nonlinear or high-order linear model reference) can be utilized effectively to control high performance aircraft such as the modified F-18 aircraft for rapid maneuvers with large changes in angle of attack.

## References

Astrom, K. J. and Wittenmark, B, 1989, Adaptive Control, Addison Wesley.

Bittanti, S., Bolzern, P. and Campi, M., 1989, "Adaptive Identification via Prediction-Error Directional Forgetting Factor Convergence Analysis," Int. J. Control, Vol. 50, pp. 2407 -2421.

Buffington, J., Sparks, A. and Banda, S., 1993, "Full Envelope Robust Longitudinal Axis Design of a Flight Aircraft with Thrust Vectoring," Proc. IEEE American Control Conf., San Francisco.

Cao, J., Arret, F., Hoffman, E. and Stalford, H., 1990, "Analytical Aerodynamic Model of a High Alpha Research Vechicle Wind-Tunnel Model," NASA CR-187469, September.

Chen, S. and Billings, S. A., 1989, "Recursive Prediction Error Parameter Estimator for Nonlinear Models," Int. J. Control, Vol. 49, pp. 569 $\sim 594$.

Cho, S. and Cho, K. R., 1994, "Adaptive Control of Highly Maneuverable High Performance Aircraft," The Korean Society for Aeronautical \& Space Sciences, Vol. 22, pp. 77~85.

Collins, D. A., 1993, "Adaptive Model Reference Control of Highly Maneuverable High Performance Aircraft," M. S. Thesis, Oregon State University.

Etkin, B., 1982, Dynamics of Flight : Stability
and Control, John Wiley \& Sons.
Garrard, G., Enns, D., and Snell, S. A., 1992, "Nonlinear Feedback Control of Highly Maneuverable Aircraft," Int. J. Control, Vol. 56. pp. 799~812.

Goodwin, G. C. and Sin, K, S., 1984, Adaptie Filtering Prediction and Control, Prentioe Hall.

Haber, R. and Unbehauen, H., 1990, "Structure Identification of Nonlinear Dynamic-A Survey on Input/Output Approaches," Automatica, Vol. 26, pp. $651 \sim 677$.

Halyo. N. and Moerder, D. D., 1989, "A Variable Gain Output Feedback Control Design Methodology," NASA CR-4226.

Leontaritis, I. J. and Billings, S., 1985, "InputOutput Parametric Models for Nonlinear System, Part I:Deterministic nonlinear Systems," Int. J. Control, Vol. 41, pp. 303-328.

Ljung, L., 1987, System Identification: Theory for the User, Prentice Hall.

McDonnel Aircraft Company, 1984, "F/A-18 Flight Control System Design Report, Vol. II : Flight Control System Analysis-Inner Loops," Report\#MDC A7813, June.

McDonnel Aircraft Company, 1982, "F/A-18 Stability and Control Data Report, Vol. I : Low Angle of Attack," Report \# MDC A7247, November.

McDonnel Aircraft Company, 1982, "F/A-18 Basic Aerodynamic Data," Report \#MDC A8575, November.

Mohler, R. R., Cho, S., Koo, C. S. and Zakrzewski, R. R., 1993, Semi-Annual Report on "Nonlinear Stability and Control Study of Highly Maneuverable High Performance Aircraft," OSUECE Report NASA 9201, Corvallis, OR, July.

Mohler, R. R., 1991, Nonlinear Systems : Vol I Dynamic and Control, Prentice Hall, 1991.

Ortega, R., Praly, L. and Landau, I. D., 1987, "Robustness of Discrete Time Direct Adaptive Controllers," IEEE Trans. Automat. Contr., Vol. 30, pp. 1179~1187.

Ostroff, A. J., 1990, "Superagility Application of Variable Gain Output Feedback Control Design Methodology," NASA High Angle of Attack Technology Conference, Hampton, Virginia.

Ostroff, A. J., 1989, "Application of VariableGain Output Feedback for High Alpha Control," ASAA Paper No. 89-3576, Guidancd, Navigation and Control, Con., Boston.

Ostroff, A. J., 1992, "High-Alpha Application of Variable-Gain Output Feedback Control," Journal of Guidance, Control, and Dynamics,

Vol. 15, pp. 491~497.
Slotine, J. J. E. and Li, W., 1991, Applied Nonlinear Control, Prentice Hall, 1991.

Sripada, N. R. and Fisher, D. G., 1987, "Improved Least Squares identification," Int. J. Control, Vol. 46, pp. 1889~1913.

